



Math Virtual Learning

Precalculus with Trigonometry

Students will solve quadratic trigonometric equations by factoring.

May 7th, 2020



Precalculus with Trigonometry

Lesson: May 7th, 2020

Objective/Learning Target:

Students will solve quadratic trigonometric equations by factoring.

Let's Get Started!

Watch Video: [Solving Quadratic Trig Equations by Factoring](#)

Example:

Find all solutions of $2 \sin^2 x - \sin x - 1 = 0$ in the interval $[0, 2\pi)$.

Begin by treating the equation as a quadratic in $\sin x$ and factoring.

$$2 \sin^2 x - \sin x - 1 = 0 \quad \text{Write original equation.}$$

$$(2 \sin x + 1)(\sin x - 1) = 0 \quad \text{Factor.}$$

Setting each factor equal to zero, you obtain the following solutions in the interval $[0, 2\pi)$.

$$2 \sin x + 1 = 0 \quad \text{and} \quad \sin x - 1 = 0$$

$$\sin x = -\frac{1}{2} \quad \sin x = 1$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6} \quad x = \frac{\pi}{2}$$

Example: Solve $5 \tan^2 \theta + \tan \theta = 0$ for $0^\circ \leq \theta < 360^\circ$

$$\tan \theta (5 \tan \theta + 1) = 0$$

Factor out the GCF

$$\tan \theta = 0$$

$$5 \tan \theta + 1 = 0$$

Set each factor to zero

$$\tan \theta = 0$$

$$\tan \theta = -\frac{1}{5}$$

Solve

Use unit circle

Use calculator (tangent is negative in Q2 and Q4)

$$\theta = 0^\circ, 180^\circ$$

$$\theta = 168.7^\circ, 348.7^\circ$$

Solve for θ

Final Answer:

$$\{0^\circ, 180^\circ, 168.7^\circ, 348.7^\circ\}$$

Example:

Find all solutions of the equation

$$2 \sin^2 \theta - 5 \sin \theta + 2 = 0.$$

Solution

Factor $2 \sin^2 \theta - 5 \sin \theta + 2 = 0$.

$$(2 \sin \theta - 1)(\sin \theta - 2) = 0$$

$$(2 \sin \theta - 1) = 0 \quad \text{or} \quad (\sin \theta - 2) = 0$$

$$\sin \theta = \frac{1}{2}$$

$$\sin \theta = 2$$

Use unit circle

$$\theta = \frac{\pi}{6} \quad \text{or} \quad \theta = \frac{5\pi}{6}$$

No solution

Sine ratio never bigger than 1

$$\theta = \frac{\pi}{6} + 2n\pi \quad \text{or} \quad \theta = \frac{5\pi}{6} + 2n\pi$$

Practice

Solve the following quadratic trigonometric equations in the interval $[0, 2\pi)$

1 $\sin^2 x - 3 \sin x + 2 = 0$

2 $3 \tan^2 x - \tan x = 0$

3 $5 \cos^2 x + 3 \cos x - 2 = 0$

4 $\sin^2 x \cos x = 4 \cos x$

Practice - ANSWERS

1 $x = \frac{\pi}{2}$

2 $x = 0, \pi, 0.32, 3.46$

3 $x = 1.16, 5.12, \pi$

4 $x = \frac{\pi}{2}, \frac{3\pi}{2}$

Additional Practice and Resources:

Additional Resource Videos:

[Solving Quadratic Trig Equations](#)

Watch the first 7:17 for more examples of today's lesson. The remaining part of the video will give you a preview for future lessons.

[Solving Trig Equations by Factoring](#)

Additional Practice:

[Solving quadratic trig equations - Kuta](#)

Try problems 1 -3, 9, 10